## AMSCO'S

## Integrated

# ALGEBRA 1 

## Ann Xavier Gantert



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## Dedication

This book is dedicated to Edward Keenan who left a profound influence on mathematics education in New York State and on the development of Amsco texts.

## Ann Xavier Gantert

This author has been associated with mathematics education in New York State as a teacher and an author throughout the many changes of the past fifty years. She has worked as a consultant to the Mathematics Bureau of the Department of Education in the development and writing of Sequential Mathematics and has been a coauthor of Amsco's Integrated Mathematics series, which accompanied that course of study.

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## PREFACE

Integrated Algebra 1 is a new text for high school algebra that continues the approach that has made Amsco a leader in presenting mathematical ideas in a contemporary, integrated manner. Over the past decades, this approach has undergone numerous changes and refinements to keep pace with ever changing technology.

This Amsco book uses an integrated approach to the teaching of high school mathematics that is promoted by the National Council of Teachers of Mathematics in its Principles and Standards for School Mathematics and mandated by the New York State Board of Regents in the New York State Mathematics Core Curriculum. This text presents a range of materials and explanations that are guidelines for achieving a high level of excellence in the study of mathematics.

In this book:
$\checkmark \quad$ The graphing calculator is introduced and used throughout the book as a routine tool in the study of mathematics. Underlying mathematical concepts and procedures are clearly presented, stressing calculator use as a learning and computational aid.
$\checkmark \quad$ The real number system is fully developed, to help students understand and correctly interpret technological limitations such as the calculator displays of rational approximations. The role of precision and accuracy, in determining acceptable computational results, is carefully explained and illustrated.
$\checkmark$ Application of algebra to the solution of problems from geometry, probability, statistics, finance, and other real-world applications is developed throughout the text.

Enrichment is stressed throughout the text and in the Teacher's Manual where multiple suggestions are given for teaching strategies, for further explorations of related topics, and for alternative assessment. The text, as well as the Manual, includes opportunities for cooperative learning, hands-on activities, extended tasks, and independent investigation. Reproducible Enrichment Activities for each chapter provide both material for review and reinforcement as well as for in-depth study.
$\checkmark \quad$ Exercises are divided into three categories. Writing About Mathematics provides questions in which students are asked to contrast, compare, evaluate, and justify their own ideas or those of others. These questions help students incorporate the tools of the performance indicators-investigate, explore, discover, conjecture, reason, justify, explain, prove, and apply-into their study of mathematics. These questions also provide a valuable source of material for classroom discussion or for inclusion in a student portfolio. Developing Skills provides routine practice exercises that enable the student and the teacher to evaluate the student's ability to both manipulate mathematical symbols and understand mathematical relationships. Applying Skills provides exercises in which the new ideas of each section, together with previously learned skills, are used to solve problems that reflect real-life situations.
$\checkmark \quad$ Conceptual understanding, procedural fluency, and problem solving, which are the primary goals of the Core Curriculum are addressed throughout the text. General concepts and principles are fully addressed and developed in detail, then further explored in the examples and exercise sections. The Procedures throughout the text explain how to perform both arithmetic and geometric processes. The Examples given in each section demonstrate problem solving approaches, often presenting alternative strategies for solution. Both routine and non-routine problems are presented.

The material in this text is intended to present basic algebra and its relationship to other branches of mathematics. The text aims at developing mathematics as a unified whole in which each branch of mathematics is integrally related. Many of the concepts presented in this text have been introduced in previous mathematics courses. The text provides the opportunity for students to review familiar material that is the foundation for the development of new topics, and presents all the material needed to develop the skills and achieve the goals suggested in the New York State Core Curriculum for Integrated Algebra.

An intent of the author was to make this text of greatest service to the average student. However, the materials for reinforcement and for enrichment that the text contains make it appropriate for varying abilities. Specifically:
$\checkmark \quad$ Concepts are carefully developed using appropriate language and mathematical symbolism.

General principles and procedures are stated clearly and concisely.
$\checkmark \quad$ Numerous solved examples serve as models for students, with detailed step-by-step explanations.
$\checkmark \quad$ Abundant and varied exercises develop skills and test understanding. Additional enrichment activities challenge the most capable student.

This text is offered so that teachers may effectively continue to help students to comprehend, master, and enjoy mathematics.

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